THEORY OF QUADRUPOLAR SUNSPOTS AND THE ACTIVE REGION OF AUGUST, 1972

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Abstract. Energy is stored when the force-free magnetic field in an active region departs from a potential field, the departure showing up as a shear in the field. As soon as the field untwists, energy will be released to produce flares. Based on this idea, we derived an analytical solution of the equation of force-free field under the assumption of a constant force-free factor, and found expressions for seven important quantities for quadrupolar sunspots: the magnetic energy of the twisted field, that of potential field, the extractable free energy ΔM, the magnetic flux, the total current, the force-free factor and the field decay factor, in terms of three observables: the field intensity, the twist angle and the distance between two spots of the same polarity. The expression for ΔM can be useful in solar prediction work.

For the active region of August, 1972, we found ΔM up to 6×10^{32} erg, sufficient to supply the energy of the observed flare activity. Observations of this active region are in good general agreement with our theoretical expectations: in the entire twisting of the quadrupolar sunspot group, each spot assumes the form of a complete spiral in the clockwise direction for each of the four spots.

1. Introduction

It is generally accepted that the free energy necessary for a solar flare is stored in magnetic fields which are distorted from their vacuum or potential field form by the presence of currents flowing in the highly conducting chromosphere and corona (van Hoven *et al.*, 1980). Since currents exist in the chromosphere above sunspots, potential theory is unreliable for calculating the magnetic fields of sunspots. However, the plasma density is very small and the magnetic field very strong, so the magnetic field there is very nearly force-free (Lüst and Schlüter, 1954), i.e., the electric current is parallel to the magnetic field elsewhere.

According to Nishi and Makita (1973), the penumbral filaments of sunspots could be considered to indicate the orientations of the transverse magnetic field lines (Tanaka and Nakagawa, 1973). Wu *et al.* (1982), Severny and Steshenko (1972), as well as Martres *et al.* (1974) have stated that the presence of complex shearing motions is the

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prerequisite for a flare in addition to the presence of large local gradients in magnetic field (Wu *et al.*, 1982; Nakagawa, 1976). Shearing motions have close relationships with the twisting of the penumbral filaments of sunspots (Ding You-ji *et al.*, 1978).

Many hypothesis have been proposed for the energy source of solar flares. Since the potential field represents the minimum energy state, with no extractable energy (Nakagawa, 1976; Tanaka and Nakagawa, 1973), the simplest source seems to be the excess energy of the force-free magnetic field over the potential field in solar active regions. Departures from a potential field are manifested by the presence of shear in magnetic fields. In particular, Wu and Hu (1982) presented a self-consistent MHD model to study the solar flare energy build-up in the solar atmosphere due to shearing motion of the foot-points of flux tubes. The stored energy can be released and flares are produced as soon as the twisting of the magnetic fields relaxes.

Based on the above idea and assuming that the force-free factor α (the ratio of current density to field strength) is constant, we have derived an analytic solution of the equation of a force-free field. The formulae of many important quantities of solar active regions have been found and applied to the cases of unipolar sunspots (Yang Hai-Shou *et al.*, 1980, 1981; Chang, 1980; Chang and Carovillano, 1981).

In this paper, we first derive the relevant formulae for quadrupolar sunspots and then apply them to the solar active region of August, 1972.

2. The Magnetic Fields of Quadrupolar Sunspots

2.1. The expressions of the magnetic fields

The fundamental equations for a force-free magnetic field are

$$\nabla \times \mathbf{B} = \alpha \mathbf{B}, \qquad \nabla \cdot \mathbf{B} = 0, \tag{1}$$

in which $\mathbf{B}$ is the magnetic field, and the force-free factor α is a scalar function. Jette (1970) showed that, when the flow velocity is zero, the necessary and sufficient condition for the maintenance of a force-free field is α = constant. We disregard the slow flow of plasma in sunspots and so take α to be constant. In this case, the solution satisfying the first equation in (1) satisfies the second equation automatically. Therefore, we need only to solve the first equation.

In cylindrical coordinates, let the origin be on the photospheric surface and the z-axis outward and perpendicular to the solar surface. Since $\mathbf{B}$ should approach zero as $z \to \infty$, we assume that $\mathbf{B}$ decays exponentially with height (Simon and Weiss, 1970; Nakagawa *et al.*, 1971), i.e., $B_z = B_{z0}(r, \theta)e^{-kz}$, in which the constant k is the decay factor of the magnetic field. Using this expression, and projecting the first equation of (1) onto the coordinate axes and simplifying, we get

$$B_r = \frac{1}{v^2}\left(\frac{\alpha}{r}\frac{\partial B_z}{\partial \theta} - k\frac{\partial B_z}{\partial r}\right), \tag{2}$$

$$B_\theta = -\frac{1}{\nu^2}\left(\frac{k}{r}\frac{\partial B_z}{\partial \theta} + \alpha\frac{\partial B_z}{\partial r}\right), \tag{3}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial B_z}{\partial r}\right)' + \frac{1}{r^2}\frac{\partial^2 B_z}{\partial \theta^2} + \nu^2 B_z = 0, \tag{4}$$

in which $\nu^2 \equiv \alpha^2 + k^2$. Equations (2)–(4) represent a force-free field which decays exponentially in the z direction.

The general solution of (4) which is finite at $r = 0$ is

$$B_z(r, \theta, z) = \sum_{m=0}^{\infty} C_m J_m(\nu r) e^{im\theta - kz}, \tag{5}$$

where the C_m's are constants of integration and the J_m's are Bessel functions of the first kind of order m. The fundamental solutions with $m = 0$ and $m = 1$ represent the magnetic fields for unipolar and bipolar sunspots, respectively (Yang Hai-shou and Zhang Hou-mei, 1980, 1981; Yang Hai-shou *et al.*, 1980, 1981). The solution with $m = 2$ is

$$B_z(r, \theta, z) = C_2 J_2(\nu r) e^{i2\theta - kz}. \tag{6}$$

The real part of this represents the magnetic field of quadrupolar sunspots:

$$B_z(r, \theta, z) = C_2 J_2(\nu r) e^{-kz} \cos 2\theta. \tag{7}$$

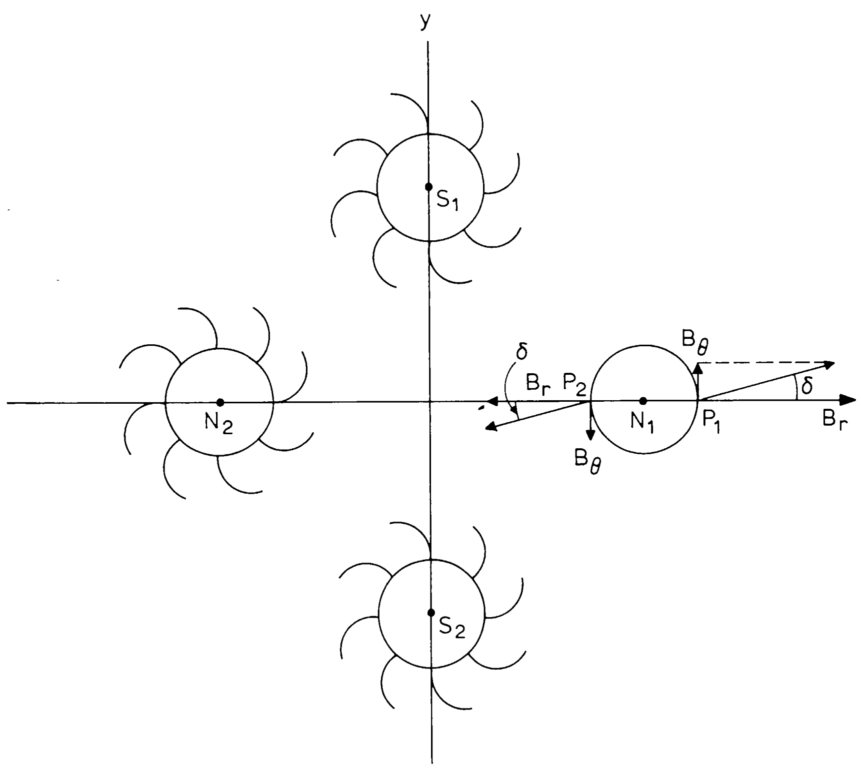

Fig. 1. Schematic diagram of the model quadrupolar sunspot. The four poles are marked N_1, N_2, S_1, S_2. The distance between S_1 and S_2 or N_1 and N_2 is defined as L. The twist angle δ is defined at pole N_1 (see text).

This may be seen as follows: At the photosphere ($z = 0$), (7) shows that $B_z = C_2 J_2(\nu r)$ for $\theta = 0$ and π. Let the point N_1 where $B_z(r, 0, 0)$ is maximum be the center of one of two sunspots with north polarity. This corresponds to the first maximum of the J_2 curve. Therefore, only the part of the J_2 curve within the first zero-point is adequate to represent the magnetic field of quadrupolar sunspots. The center of the other sunspot of north polarity is N_2 at $\theta = \pi$, also corresponding to the first maximum of the J_2 curve. For $\theta = \pi/2$ or $3\pi/2$, $B_z = -C_2 J_2(\nu r)$. At $\nu r = 3.054$, $B_{z\,\max} = -C_2 J_{2\,\max}$, corresponding to the two sunspots of south polarity. The four sunspots N_1, N_2, S_1, and S_2 of the quadrupolar sunspot group are shown in Figure 1.

Quadrupolar sunspot groups have higher probabilities of producing solar proton flares than unipolar and bipolar sunspots (Shi Zhong-xian *et al.*, 1975). Therefore, it is worthwhile to study quadrupolar sunspots in more detail.

The constant C_2 in (7) can be found as follows: On the photospheric surface ($z = 0$), the center of each sunspot is at the point where the absolute value of B_z is maximum: $B_{z\,\max} \equiv B_0$. From (7), this corresponds to $J_2(\nu L/2) = J_{2\,\max} = J_2(3.054) = 0.4865$. We may also define the boundary of the sunspot group to be where $B_z = 0$, corresponding to $J_2(\nu r_0) = 0$ or $\nu r_0 = 5.136$, at radial distance r_0. Thus

$$r_0 = 0.84L. \tag{8}$$

In (7), taking $z = 0$, $\theta = 0$, and $r = L/2$, we find $B_z(L/2, 0, 0) = C_2 J_{2\,\max} = 0.4865 C_2 \equiv B_0$. Thus

$$C_2 = 2.055 B_0. \tag{9}$$

Substituting (9) into (7), we get (12) below and then substituting (12) into (2) and (3) we find (10) and (11):

$$B_r = -\frac{2.055 B_0}{\nu^2} e^{-kz} \left\{ \frac{2\alpha}{r} J_2(\nu r) \sin 2\theta + k\left[\nu J_1(\nu r) - \frac{2}{r} J_2(\nu r) \right] \cos 2\theta \right\}, \tag{10}$$

$$B_\theta = -\frac{2.055 B_0}{\nu^2} e^{-kz} \left\{ -\frac{2k}{r} J_2(\nu r) \sin 2\theta + \alpha\left[\nu J_1(\nu r) - \frac{2}{r} J_2(\nu r) \right] \cos 2\theta \right\}, \tag{11}$$

$$B_z = 2.055 B_0 e^{-kz} J_2(\nu r) \cos 2\theta. \tag{12}$$

These are the required expressions of the components of magnetic field for quadrupolar sunspots.

In order to determine the constants ν, k, and α, the four sunspots (N_1, N_2, S_1, and S_2) are chosen to be circular with their centers on the coordinate axes (Figure 1) at equal distances $L/2$ from the origin on the photospheric plane ($z = 0$). Due to horizontal motions in the photosphere (Moore *et al.*, 1980), the penumbral filaments of each spot are twisted as shown in Figure 1 (the filaments of spot N_1 are omitted for simplicity). The angle δ through which the transverse field line is twisted can be found as follows: Substituting $z = 0$ and $\theta = 0$, $\pi/2$, or $3\pi/2$ into (10) and (11), we get

$$\delta = \tan^{-1}\left[\frac{B_\theta}{B_r}\right]_{z=0} = \tan^{-1}\frac{\alpha}{k}, \tag{13}$$

which is seen to be independent of r and it implies sunspots having uniform twists. E.g., δ has the same value at two points P_1 and P_2, the intersections of the x-axis with the boundary of the umbra of spot N_1. It is similar for the other three spots.

The solar active region of August 1972 provides a good example of a quadrupolar sunspot group (Figure 2). This consisted of four main spots (Ding You-ji *et al.*, 1977): A, O, B and S, corresponding to our spots N_1, N_2, S_1, and S_2, respectively. Ding You-ji *et al.* have reported that all the large (2B and 3B) flares in this active region were closely related to two spots: (i) Spot $B(S_1)$ – on August 1, the penumbral filaments within a semi-circle on its west and south showed a consistent spiral pattern (clockwise, i.e.,

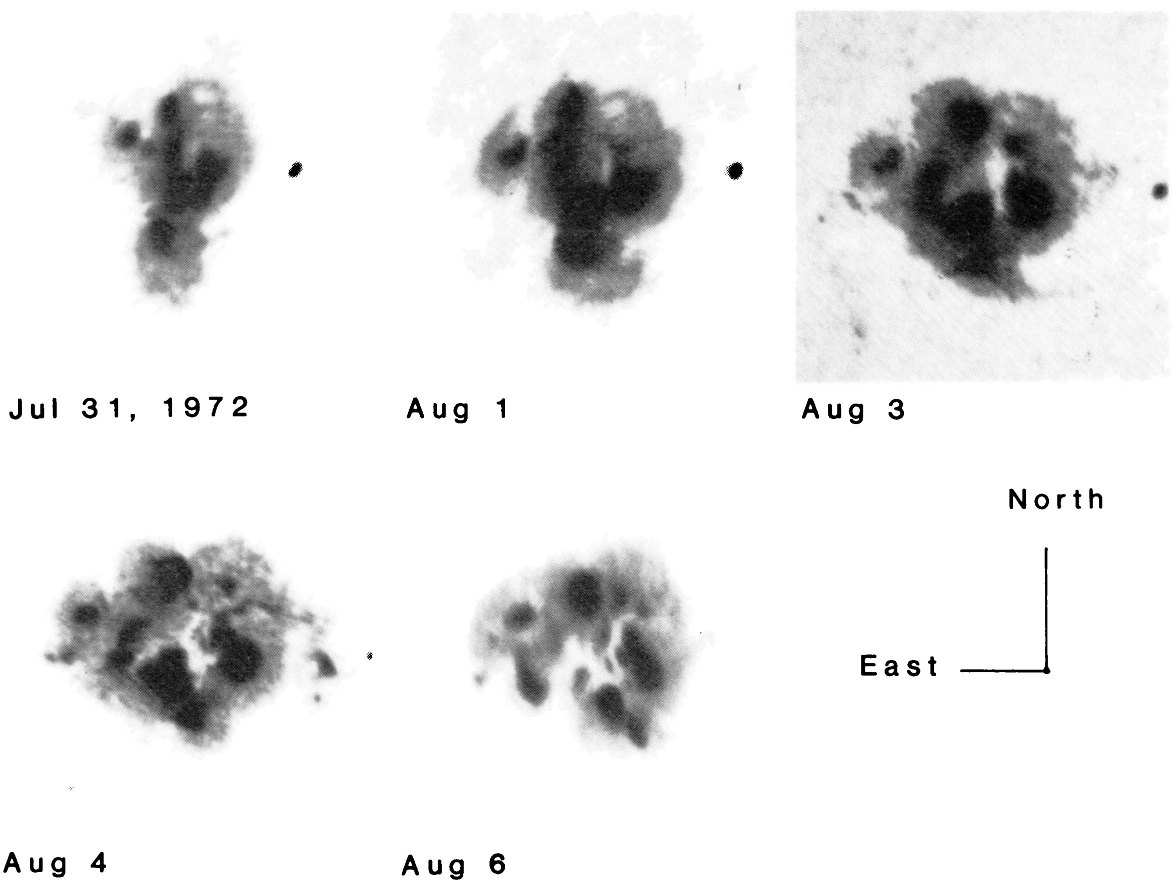

Fig. 2a. White light photographs of solar active region McMath 11976 taken at Mr. Wilson Observatory (courtesy of Dr. R. Howard).

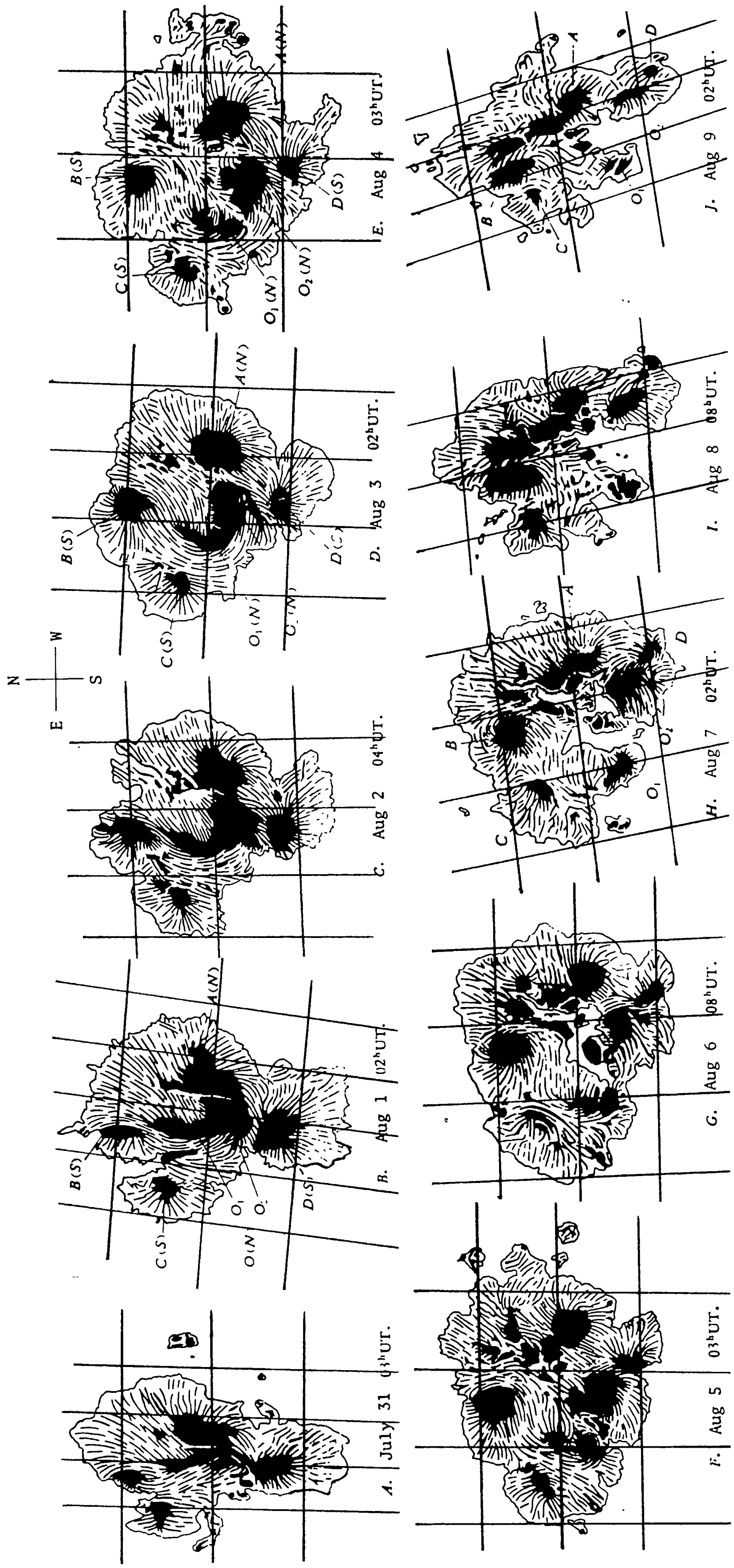

Fig. 2.b. Sketches of penumbral fine structure around the sunspots of McMath 11976 (Ding *et al.*, 1977).

$\delta < 0$). From August 2 to August 5, the spiral pattern continually enlarged and the radial pattern diminished continually. On August 6 and 7, a consistent, clockwise ($\delta < 0$) spiral pattern was present on all sides of spot B. (ii) Spot $O(N_2)$ – on August 2, this began to assume the form of a complete clockwise spiral (again, $\delta < 0$). On August 3 and 4, it gradually broke up, but the tendency of spot O_2 to become more circular and spiral-like was quite obvious. The situations were similar for the other two spots. This is in good general agreement with our theoretical expectations: in the entire twisting of the quadrupolar sunspot group, each spot assumes the form of a complete spiral in the clockwise direction, i.e., the same $\delta < 0$ for each of the four spots.

From (2)–(4) and (13), we get

$$v = \sqrt{\alpha^2 + k^2} = k \sec \delta, \tag{14}$$

while from (8), (13), (14) and $vr_0 = 5.136$, if follows that

$$v = \frac{6.114}{L} \quad \text{cm}^{-1}, \tag{15}$$

$$k = \frac{6.114}{L} \cos \delta \, \text{cm}^{-1}, \tag{16}$$

$$\alpha = \frac{6.114}{L} \sin \delta \, \text{cm}^{-1}. \tag{17}$$

2.2. The magnetic energy

The magnetic energy stored in the magnetic field of the quadrupolar sunspot group can be calculated from (10)–(12), with (15)–(17), as

$$\begin{aligned} M &= \iiint \frac{|\mathbf{B}^2|}{8\pi} \, \mathrm{d}V \\ &= \frac{4.223 B_0^2}{8\pi} \int_0^\infty e^{-2kz} \, \mathrm{d}z \left\{ \frac{4}{v^2} \int_0^{2\pi} \mathrm{d}\theta \int_0^{r_0} \frac{1}{r} J_2^2(vr) \, \mathrm{d}r + \right. \\ &\quad \left. + \int_0^{2\pi} \cos^2 2\theta \, \mathrm{d}\theta \int_0^{r_0} \left[J_1^2(vr) + J_2^2(vr) - \frac{4}{vr} J_1(vr) J_2(vr) \right] r \, \mathrm{d}r \right\} = \\ &= 0.003\,51 B_0^2 L^3 \sec \delta \text{ erg}. \end{aligned} \tag{18}$$

When the twisting angle δ of the field lines at each spot is zero, the magnetic energy of the potential field is

$$M_p = 0.003\,51 B_0^2 L^3 \text{ erg}. \tag{19}$$

The extractable free energy of twisting is thus

$$\Delta M = M - M_p = 0.003\,51 B_0^2 L^3 (\sec \delta - 1)\ \text{erg}. \tag{20}$$

2.3. The total magnetic flux and electric current

On the photospheric surface ($z = 0$), (12) shows that $B_z > 0$ where $-\pi/4 < \theta < \pi/4$ or $3\pi/4 < \theta < 5\pi/4$ (marked by the plus sign in Figure 3), while $B_z < 0$ where

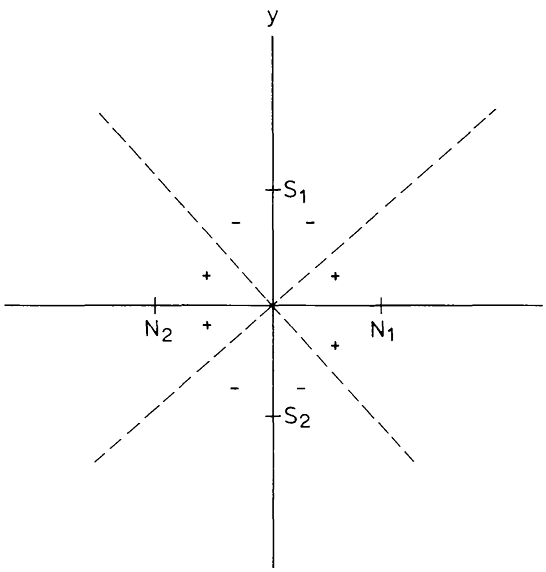

Fig. 3. The distribution of the sign of B_z in the model.

$\pi/4 < \theta < 3\pi/4$ or $5\pi/4 < \theta < 7\pi/4$ (marked by the minus sign in Figure 3). However, we define the total magnetic flux Φ as

$$\Phi = \iint |B_z|_{z=0}\, \mathrm{d}S = 8 \int_{\theta=0}^{\pi/4} \int_{r=0}^{r_0} |B_z|_{z=0}\, r\, \mathrm{d}\theta\, \mathrm{d}r = 0.882 B_0 L^2\ \text{G cm}^2. \tag{21}$$

From the first equation of (1) and Ampére's Law, the total electric current I through the sunspot group can be calculated as

$$I = \iint |j_z|_{z=0}\, \mathrm{d}S = \frac{c\alpha}{4\pi} \iint |B_z|_{z=0}\, \mathrm{d}S = \frac{10\alpha}{4\pi}\, \Phi = 4.30 B_0 L \sin\delta\ \text{amp}. \tag{22}$$

3. The Solar Active Region of August, 1972

The above formulae are now applied to the solar active region of August, 1972. From the morphological maps of this active region (Ding You-ji *et al.*, 1977) and the daily white-light fine-structure photographs we measured for each day the distance between

TABLE I

Physical quantities of active region McMath 11976 (July 31–August 9, 1972)

Date 1972	B_0 (G)	$-\delta$ (°)	L (cm)	$-\alpha$ (cm^{-1})	M (erg)	M_p (erg)	ΔM (erg)	Φ (G cm^2)	I (amp)
July 31	2700	20	2.70×10^9	7.74×10^{-10}	5.36×10^{32}	5.04×10^{32}	0.32×10^{32}	1.74×10^{22}	10.72×10^{12}
Aug. 1	2900	18	3.26	5.80	10.75	10.23	0.52	2.72	12.56
Aug. 2	3050	26	3.44	7.79	14.79	13.29	1.50	3.18	19.78
Aug. 3	2500	25	3.42	7.56	9.68	8.78	0.90	2.58	15.54
Aug. 4	2700	39	3.45	11.15	13.52	10.51	3.01	2.84	25.20
Aug. 5	2700	46	3.77	11.67	19.74	13.71	6.03	3.38	31.48
Aug. 6	2500	44	3.33	12.75	11.26	8.10	3.16	2.44	24.86
Aug. 7	2800	48	3.38	13.44	15.88	10.63	5.25	2.82	30.24
Aug. 8	2300	21	3.58	6.12	9.13	8.52	0.61	2.60	12.68
Aug. 9	2600	3	3.59	0.89	10.99	10.98	0.01	2.96	2.10

the N_1 and N_2 spots, as well as that between the S_1 and S_2 spots, and then take their mean value L; also we measured the twist angle δ (average measurements over several directions). The magnetic field data were taken from the *Bulletin on Sunspot Magnetic Fields* published by the Academy of Sciences of the U.S.S.R.

The relevant data and calculated results are given in Table I. The minus sign for δ and α is indicating that the twisting of the penumbral filaments is clockwise, i.e., $\delta < 0$. Table I shows that the total electric current I is very large, up to 3.1×10^{13} amp. It may lead to certain MHD instability (tearing mode) in the surrounding atmosphere, relaxing the twists in the magnetic field and releasing the free-energy ΔM.

Figure 4 shows the daily variations of the extractable free energy ΔM and the total magnetic energy M. It can be seen from Table I and Figure 4 that a large amount of extractable energy ΔM, up to $(5.23–6.03) \times 10^{32}$ erg is stored in the field in departures from the potential field in a relatively small area of 10^{19} cm^2. We can see from Figure 4 that ΔM increased to 1.5×10^{32} erg from July 31 to August 2; then with the eruptions of 5 small flares of importance 1 and a 2B flare on August 2, ΔM decreased to a minimum on August 3. After that, ΔM increased to a maximum of 6.03×10^{32} erg and then

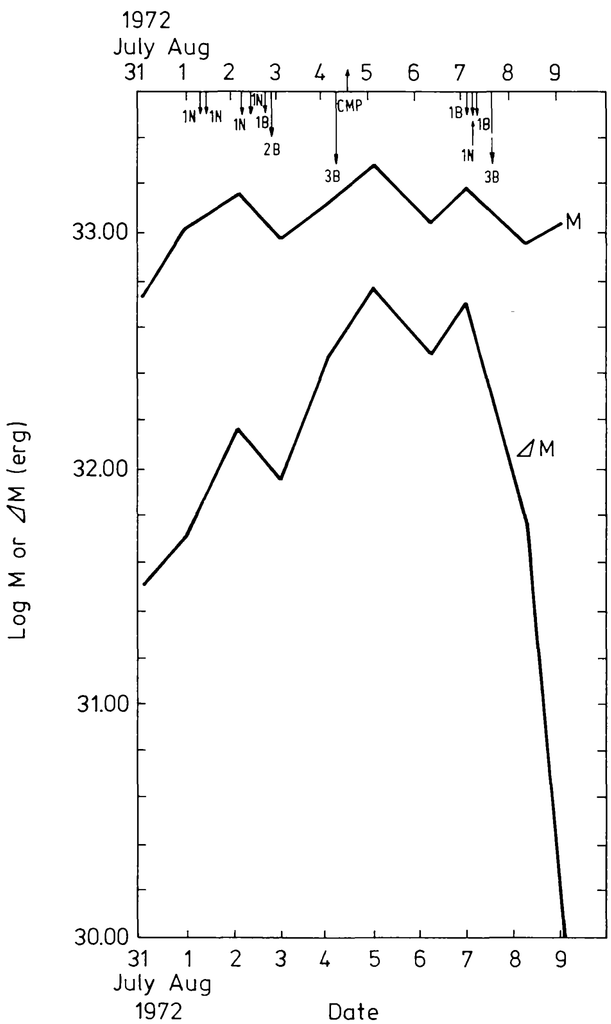

Fig. 4. Variation of calculated total energy M, and extractable free energy ΔM. Times and sizes of flares are indicated at the top as is the time of central meridian passage of the active region (CMP).

decreased again after the eruption of a big 3B flare on August 4. Passing a minimum on August 6, ΔM increased to another maximum of 5.25×10^{32} erg on August 7; then after the eruptions of three small flares of importance 1 and another big 3B flare on August 7, ΔM decreased monotonically. M had similar variations with time as ΔM.

The increase of ΔM over July 31–August 2, the second and third increase on August 5 and 7, respectively, and the subsequent decreases are all caused mainly by the changes in the twist angle δ (Table I, column 3) and additionally by the changes in B_0 (column 2) and L (column 4).

The calculated flux Φ is between 1.74×10^{22} and 3.38×10^{22} G cm^2, which is just the sort of flux typical of a large sunspot group, since a large sunspot may contain 3×10^{22} G cm^2, a typical active region contains 10^{22} G cm^2 and a small active region contains about 10^{21} G cm^2 (Galloway and Weiss, 1981). However, due to the fact that the actual structure of this sunspot group is more complex than that of our model, the values of Φ may be somewhat underestimated.

Tanaka and Nakagawa (1973) studied this active region in the context of evolution of constant-α force-free magnetic fields, and showed that the excess magnetic accumulated in a force-free field by relative motions of sunspots can account for the energy released in the flares of August 1972. However, they treated this sunspot group as a bipolar group and so could not account for the entire twisting pattern of the group. But we take this sunspot group as a quadrupolar group, which is more realistic and closer to the actual situation. As stated in Section 2, our model is in good general agreement with observations. Our theoretical expectations account well for the entire twsiting of this quadrupolar sunspot group, in which each spot assumes the form of a complete spiral. We have calculated seven important quantities of this sunspot group as shown by Table I and formula (23). The peak value of free energy estimated by Tanaka and Nakagawa is 5.7×10^{32} erg, while our value, calculated with the more accurate quadrupolar theory, is 6.03×10^{32} erg (Table I). The near equality of these two values shows that for the calculation of magnetic energy, the simpler bipolar theory is adequate.

4. Conclusions

(1) We have derived the following expressions for the various quantities of a quadrupolar sunspot group based on the force-free field theory:

$$
\begin{array}{llll}
\text{field decay factor} & k = (6.114 \cos \delta)/L & (\text{cm}^{-1}), & \\
\text{force-free factor} & \alpha = (6.114 \sin \delta)/L & (\text{cm}^{-1}), & \\
\text{magnetic flux} & \Phi = 0.882 B_0 L^2 & (\text{G cm}^2), & \\
\text{electric current} & I = 4.30 B_0 L \sin \delta & (\text{amp}), & (23) \\
\text{energy of twisted field} & M = 0.00351 B_0^2 L^3 \sec \delta & (\text{erg}), & \\
\text{energy of potential field} & M_p = 0.00351 B_0^2 L^3 & (\text{erg}), & \\
\text{extractable free energy} & M = 0.00351 B_0^2 L^3 (\sec \delta - 1) & (\text{erg}), &
\end{array}
$$

in which the maximum field intensity B_0, the twist angle δ of penumbral filaments, and the distance L between two spots of the same polarity are all observables.

(2) The active region of August, 1972 stored between 0.01×10^{32} and 6.03×10^{32} erg of extractable free energy. As the sunspot group untwisted, the energy was released and flares were produced. This shows that the mechanism for the flare activity in this active region is a transition of the force-free field of the sunspot group from a state of higher energy (larger δ) to a state of lower energy (smaller δ).

(3) Formula (20) may be useful in solar prediction work. If the calculated $\Delta M \sim 10^{28}$–5×10^{30} erg, then subflares may occur; if $\Delta M \sim 10^{30}$–10^{31} erg, then ordinary flares; if $\Delta M > 10^{32}$ erg, then large flares. The validity of this statement awaits further check in solar prediction work.

(4) This study provides an example which shows that for the calculation of the build-up of magnetic energy, the simpler bipolar theory of Tanaka and Nakagawa (1973) is a good approximation, which can be applied to calculate ΔM later on.

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